The dispersive contribution of $\rho(1450, 1700)$ decays and $X(1576)$

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We study whether the broad enhancement $X(1576)$ arises from the final state interaction (FSI) of $\rho(1450, 1700) \to \rho^+\rho^- \to K^+K^-$ decays. We consider both the absorptive and dispersive contribution of the above amplitudes since the intermediate states are very close to $\rho(1450, 1700)$. The same mechanism leads to a similar enhancement around 1580 MeV in the $\pi^+\pi^-$ spectrum in the $J/\psi \to \pi^0\pi^+\pi^-$ channel, which can be used to test whether $X(1576)$ can be ascribed to the FSI effect of $\rho(1450, 1700) \to \rho^+\rho^-$.

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I. INTRODUCTION

A broad enhancement $X(1576)$ with $J^{PC}I^G = 1^{--}1^+$ was observed by BES Collaboration in K^+K^- invariant mass spectrum in the $J/\psi \to \pi^0K^+K^-$ channel [1]. Its resonance parameters are $m = (1576^{+49}_{-55}(\mathrm{stat})^{+98}_{-91}(\mathrm{syst})) - i(409^{+11}_{-12}(\mathrm{stat})^{+32}_{-67}(\mathrm{syst}))$ MeV. The branching ratio is $B[J/\psi \to X(1576)\pi^0] \cdot B[X(1576) \to K^+K^-] = (8.5 \pm 0.6^{+2.7}_{-3.6}) \times 10^{-4}$. Its extremely large width around 800 MeV motivated theoretical explanations such as a $K^*(892) - \kappa$ molecular state [2], tetraquark [3, 4], diquark-antidiquark bound state [5, 6], a composition of $\rho(1450)$ and $\rho(1700)$ [7].

Since there are two broad overlapping resonances $\rho(1450)$ and $\rho(1700)$ with the same quantum number around 1600 MeV, we investigated whether such a broad signal could be produced by the final state interaction (FSI) [8] effect in our previous work [9]. We noticed that the interference effect of $\rho(1450, 1700)$ could produce an enhancement around 1540 MeV with the opening of the $\rho\rho$ channel, similar to the cusp effect discussed in Ref. [10]. However, the branching ratio $B[J/\psi \to \pi^0\rho(1450, 1700)] \cdot B[\rho(1450, 1700) \to K^+K^-]$ from the FSI effect was far less than the experimental data. It's important to point out that we considered only the contribution of the absorptive part in Ref. [9].

Recently Meng and Chao explored the possible assignment of $X(3872)$ as the χ'_{c1} candidate [11]. They found the dispersive part of the FSI amplitude contributes more importantly to the hidden charm decay width of $X(3872)$ than the imaginary part derived in Ref. [12] because the intermediate states $D^0\bar{D}^{0*}$ and D^-D^{+*} lie very close to $X(3872)$. Motivated by the above observation, we investigate the potential role of the dispersive contribution of FSI since the $\rho\rho$ intermediate states are rather close to $\rho(1450, 1700)$.

This paper is organized as follows. The formulation of $\rho(1450, 1700) \to \rho^+\rho^- \to K^+K^-$ by exchanging $K^{0(*)}$ is presented in Section II and our numerical result and discussion in Section III.

II. FORMULATION

As shown in Fig. 1, we focus on the FSI of the $\rho\rho$ intermediate states through the exchange of the $K^{0(*)}$ meson: $\rho(1450, 1700) \to \rho^+\rho^- \to K^+K^-$.

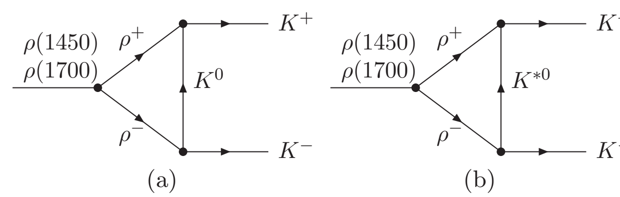

FIG. 1: The decay of $\rho(1450, 1700) \to K^+K^-$ through the $\rho^\pm$ pair.

In order to derive the absorptive amplitude, we introduce the effective Lagrangians

$$
\begin{aligned}
\mathcal{L}_{V_1 \to P_1P_2} &= ig_1(P_1 \overleftrightarrow{\partial} P_2)V^\nu, &(1)\\
\mathcal{L}_{V_1 \to V_2P_1} &= g_1\epsilon_{\mu\nu\alpha\beta}V_1^\mu\partial^\nu P_1\partial^\beta V_2^\alpha, &(2)\\
\mathcal{L}_{V_1 \to V_2V_3} &= ig_2\Big\{V_1^\mu(\partial_\mu V_2^\nu V_{3\nu} - V_2^\nu\partial_\mu V_{3\nu}) \\
&\quad +(\partial_\mu V_{1\nu}V_2^\nu - V_{1\nu}\partial_\mu V_2)V_3^\mu \\
&\quad +V_2^\mu(V_1^\nu\partial_\mu V_{3\nu} - \partial_\mu V_{1\nu}V_3^\nu)\Big\}, &(3)
\end{aligned}
$$

where g_i's denote the coupling constants. P_i and V_i respectively denote the pseudoscalar and vector fields.

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A. Absorptive contribution

Using the Cutkosky cutting rule, one obtains the the absorptive contribution to the process of $\rho(1450,1700) \to \rho^+(p_1)\rho^-(p_2) \to K^+(p_3)K^-(p_4)$ with the exchanged mesons K^0 and K^{*0}

$$\begin{aligned}
&\mathbf{Abs}^{(a)}[\rho^+\rho^-, K^0] \\
&= \frac{1}{2}\int \frac{d^3p_1}{(2\pi)^3 2E_1}\frac{d^3p_2}{(2\pi)^3 2E_2}(2\pi)^4\delta^4(P-p_1-p_2) \\
&\times\Big\{ig_{\rho(1450)\rho\rho}[\epsilon\cdot(p_1-p_2)g_{\mu\nu}-\epsilon_\mu(2p_1+p_2)_\nu \\
&+\epsilon_\nu(2p_2+p_1)_\mu]\Big\}[ig_{\rho KK}(q_\alpha+p_{3\alpha})] \\
&\times[ig_{\rho KK}(q_\beta-p_{4\beta})]\Big[-g^{\nu\alpha}+\frac{p_1^\nu p_1^\alpha}{m_\rho^2}\Big] \\
&\times\Big[-g^{\mu\beta}+\frac{p_2^\mu p_2^\beta}{m_\rho^2}\Big]\Big[\frac{i}{q^2-m_K^2}\Big]\mathcal{F}^2(m_K,q^2), \qquad (4)
\end{aligned}$$

and

$$\begin{aligned}
&\mathbf{Abs}^{(b)}[\rho^+\rho^-, K^{*0}] \\
&= \frac{1}{2}\int \frac{d^3p_1}{(2\pi)^3 2E_1}\frac{d^3p_2}{(2\pi)^3 2E_2}(2\pi)^4\delta^4(P-p_1-p_2) \\
&\times\Big\{ig_{\rho(1450)\rho\rho}[\epsilon\cdot(p_1-p_2)g_{\mu\nu}-\epsilon_\mu(2p_1+p_2)_\nu \\
&+\epsilon_\nu(2p_2+p_1)_\mu]\Big\}[ig_{\rho KK^*}\epsilon_{\alpha\beta\kappa\gamma}p_1^\alpha q^\kappa] \\
&\times\Big[ig_{\rho KK^*}\epsilon_{\xi\lambda\delta\zeta}p_2^\xi q^\delta\Big]\Big[-g^{\nu\beta}+\frac{p_1^\nu p_1^\beta}{m_\rho^2}\Big] \\
&\times\Big[-g^{\mu\lambda}+\frac{p_2^\mu p_2^\lambda}{m_\rho^2}\Big]\Big[-g^{\gamma\zeta}+\frac{q^\gamma q^\zeta}{m_{K^*}^2}\Big] \\
&\times\Big[\frac{i}{q^2-m_{K^*}^2}\Big]\mathcal{F}^2(m_{K^*},q^2). \qquad (5)
\end{aligned}$$

In the above expressions, $\mathcal{F}(m_i,q^2)$ etc denotes the form factors which compensate the off-shell effects of mesons at the vertices and are written as [13, 14]

$$\mathcal{F}(m_i,q^2) = \left(\frac{\Lambda^2-m_i^2}{\Lambda^2-q^2}\right)^n, \qquad (6)$$

where Λ is a phenomenological parameter. As $q^2 \to 0$ the form factor becomes a number. If $\Lambda \gg m_i$, it becomes unity. As $q^2 \to \infty$, the form factor approaches to zero. As the distance becomes very small, the inner structure would manifest itself and the whole picture of hadron interaction is no longer valid. Hence the form factor vanishes and plays a role to cut off the end effect. The expression of Λ is [13]

$$\Lambda(m_i) = m_i + \alpha\Lambda_{QCD}, \qquad (7)$$

where m_i denotes the mass of exchanged meson and α is a phenomenological parameter. Although we use $\Lambda_{QCD} = 220$ MeV, the range of Λ_{QCD} can be taken into account through the variation of the parameter α. In this work, we adopt the monopole form factor $\mathcal{F}(m_i,q^2) = (\Lambda^2-m_i^2)/(\Lambda^2-q^2)$, where α is of order unity and its range is around $0.8 < \alpha < 2.2$ [13].

B. Dispersive contribution

As the bridge between the dispersive part of FSI amplitude and the absorptive part, the dispersion relation is

$$\mathbf{Dis}\mathcal{M}(m_X) = \frac{1}{\pi}\int_{s_0}^{\infty}\frac{\mathbf{Abs}\mathcal{M}(s)}{s-m_X^2}ds, \qquad (8)$$

with

$$\begin{aligned}
\mathbf{Abs}\mathcal{M}(s) \;=\; &\Big\{\mathbf{Abs}^{(a)}[\rho^+\rho^-,K^0] \\
&+\mathbf{Abs}^{(b)}[\rho^+\rho^-,K^{*0}]\Big\}\exp(-\beta|\mathbf{k}|^2),
\end{aligned}$$

where $\mathbf{k} = \sqrt{s/4-m_\rho^2}$ is the three momentum of $\rho^\pm$ in the rest frame of $\rho(1450,1700)$. The exponential reflects the dependence of the interaction between $\rho(1450,1700)$ and $\rho^\pm$ on $\mathbf{k}$, which also plays the role of the cutoff. The factor β is related to the radius of interaction R by $\beta = R/6$ [15].

Using the same formalism, we calculate the decay amplitude of $\rho(1450,1700) \to \rho^+\rho^- \to \pi^+\pi^-$ as depicted in Fig. 2.

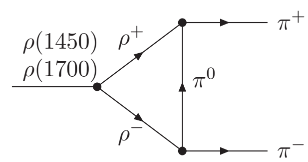

FIG. 2: The decay of $\rho(1450,1700) \to \pi^+\pi^-$ through the $\rho^\pm$ pair.

III. RESULTS AND DISCUSSIONS

Using $\Gamma(\phi \to K^+K^-) = 2.09$ MeV [16], we obtain $g_{\phi K^+K^-} = 5.55$. In the limit of SU(3) symmetry, we take $\sqrt{2}g_{\rho^0K^\pm K^\mp} = g_{\phi K^\pm K^\mp}$. $g_{\rho^\pm K^\mp K^{*0}} = 6.48$ GeV^{-1} [17]. $g_{\rho(1450)\rho^+\rho^-} = 1.53$ and $g_{\rho^+\pi^0\pi^+} = g_{\rho^-\pi^0\pi^-} = 11.5$ [16].

In Fig. 3, we show the dependence of the width of $\rho(1450,1700) \to \rho^+\rho^- \to K^+K^-$ on the mass of $\rho(1450,1700)$ with the typical parameters $\alpha = 1.0, 1.5, 2.0$ and $\beta = 0.2, 0.4, 0.8$ GeV^{-2} [15]. 4 is the dependence of the width of $\rho(1450,1700) \to \rho^+\rho^- \to \pi^+\pi^-$ on the mass of $\rho(1450,1700)$ with several typical values.

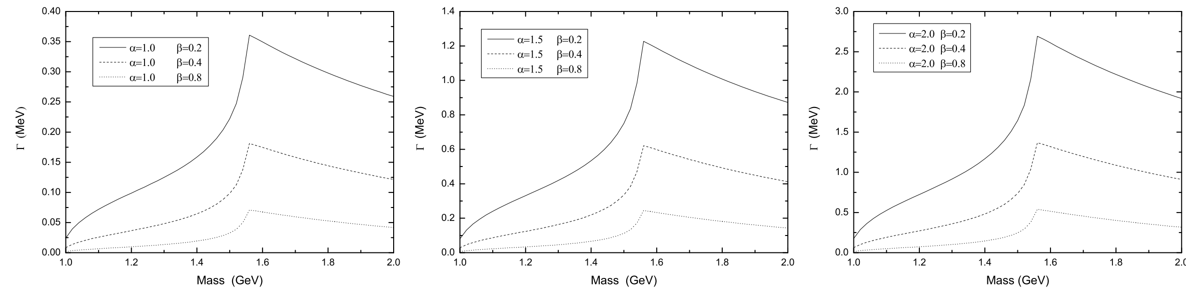

FIG. 3: The dependence of the width of $\rho(1450, 1700) \to K^+K^-$ on the the mass of $\rho(1450, 1700)$ with several typical values of α and β.

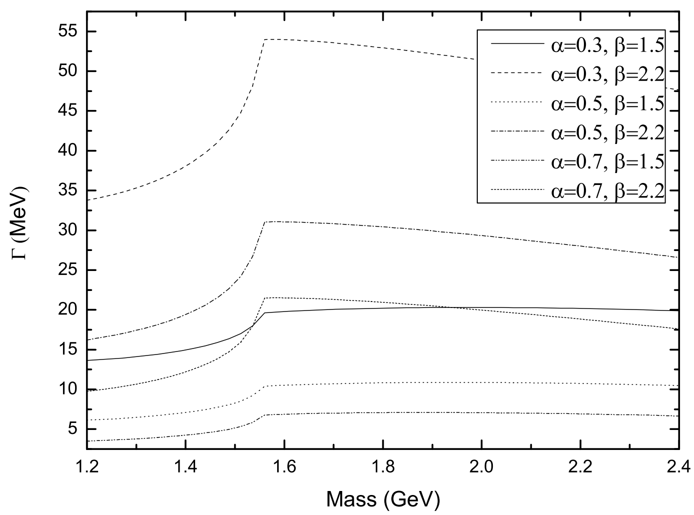

FIG. 4: The dependence of the width of $\rho(1450, 1700) \to \pi^+\pi^-$ on the the mass of $\rho(1450, 1700)$ with several typical values of α and β.

In this short note, we revisit the final state interaction of the $\rho(1450, 1700) \to \rho^+\rho^-$ decay. Different from our former work [9], we consider the additional contribution from the dispersive part. Our result shows that there exists an enhancement around 1580 MeV as shown in Fig. 3. Such an enhancement occurs with the opening of the $\rho\rho$ channel.

The decay width of $\rho(1450, 1700) \to K^+K^-$ from the FSI effect is a few MeV only. If the width of $\rho(1450, 1700)$ is 300 MeV, the branching ratio of $\rho(1450, 1700) \to A + B \to K^+K^-$ is about 10^{-2}. With $B[J/\psi \to \pi + \rho(1450, 1700)]$ roughly around 10^{-3} [16], $B[J/\psi \to \pi^0 + \rho(1450, 1700)] \cdot B[\rho(1450, 1700) \to AB \to K^+K^-]$ is about 10^{-5}. The dispersive contribution enhances the branching ratio of $\rho(1450, 1700) \to K^+K^-$ by two orders than that in Ref. [9]. However, such a ratio is still far less than experimental value $B[J/\psi \to \pi + X(1576)] \cdot B[X(1576) \to K^+K^-] = (8.5 \pm 0.6^{+2.7}_{-3.6}) \times 10^{-4}$, although the K^+K^- spectrum from the FSI effect of $\rho(1450, 1700)$ decays mimics the observed broad spectrum from BES's measurement.

Throughout our calculation, we ignored the direct coupling between $\rho(1450, 1700)$ and $K\bar{K}$. Recently, Li argued that $\rho(1450, 1700)$ can have strong coupling with K^+K^- at the tree level [7]. Adding this contribution certainly increases the branching ratio. However, the experimental upper limit of $B[\rho(1450) \to K\bar{K}]$ is 1.6×10^{-3} and $K\bar{K}$ is not one of the dominant decay modes of $\rho(1700)$ [16]. Clearly, future BESIII high-statistics data around 1.6 GeV in the $K\bar{K}$ channel will be very helpful in the clarification of $X(1576)$. We also calculate the decay amplitude of $\rho(1450, 1700) \to \rho^+\rho^- \to \pi^+\pi^-$ using the same

technique. There exists one similar enhancement around 1580 MeV, which is shown in Fig. 4. This enhancement will be useful to test if X(1576) arises from the FSI effect.

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